

# A COMPACT WAVEGUIDE 'RESOLVER' FOR THE ACCURATE MEASUREMENT OF COMPLEX REFLECTION COEFFICIENTS USING THE 6 PORT MEASUREMENT CONCEPT

G.P.Riblet  
Microwave Development Labs., Inc.  
11 Michigan Drive, Natick, Ma. 01760

## ABSTRACT

A compact waveguide network is described which allows one to accurately determine complex reflection coefficients over waveguide bandwidths from the measurement of two referenced power levels using simple formulae.

### Introduction

Recently, Engen has described a number of networks which can be used in either stripline or waveguide media to make very accurate measurements of complex reflection coefficients via the measurement of the power levels at four detectors.<sup>1</sup> These networks can be used over full waveguide bandwidths in waveguide and multi-octave bandwidths in stripline. Nevertheless these networks have some significant drawbacks from the economic point of view: a) they contain many components which makes them large and expensive - at least in waveguide, b) an extensive calibration procedure must be employed before measurements can start, c) the required quantity  $\Gamma = |\Gamma|e^{j\theta}$  and the measured powers aren't related by simple formulas thereby increasing computation time. Consequently, it would be desirable in waveguide to have a network which

- 1) is compact.
- 2) is calibrated using a very simple calibration procedure.
- 3) relates  $\Gamma$  to measured powers using simple formulae.

In this contribution a network which satisfies these requirements is described. It is useful for VSWR's  $< 2/1$  which includes the cases of most practical interest.

### Theory of an Optimum 5 Port

Engen has introduced a diagram in the complex  $\Gamma$  plane which is quite useful for determining the circumstances under which  $\Gamma$  can be unambiguously and accurately determined from the measurement of the power levels at a number of detectors.<sup>2</sup> One of these detectors is connected to a directional coupler and serves to monitor the incident power. Each additional detector can be represented by a point  $q_i$  in the  $\Gamma$  plane. Engen has suggested that the optimum situation in terms of accuracy arises when there are three additional detectors and the points  $q_i$  are separated by  $120^\circ$  on a circle with its center at the origin and diameter  $> 1$ . The device will have a total of 6 ports when the signal port and a port for the device under test are included. He has, however, pointed out that in principle 5 ports are sufficient. We will be considering here a 5 port version of the 6 port scheme (see figure 1 for a schematic diagram).

It will be argued that for small VSWR's ( $\rho < 2/1$ ) the optimum 5 port network has two additional detectors with  $q$  values  $q_1$  and  $q_2$  separated by  $90^\circ$  on the unit circle. In this case  $\Gamma$  will be uniquely determined by the intersection of 2 circles of known radius as indicated in figure 2. Since the circles intersect at nearly  $90^\circ$  independent of phase within the hatched area, the accuracy will be nearly independent of the

actual reflected phase  $\theta$ .

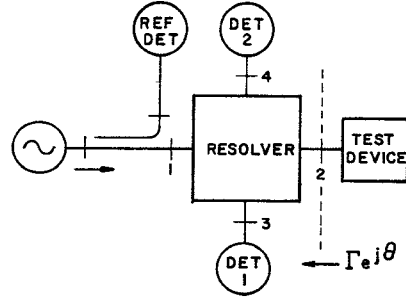


Fig.1 Schematic diagram of an experimental arrangement for the measurement of  $\Gamma$  with a five-port.

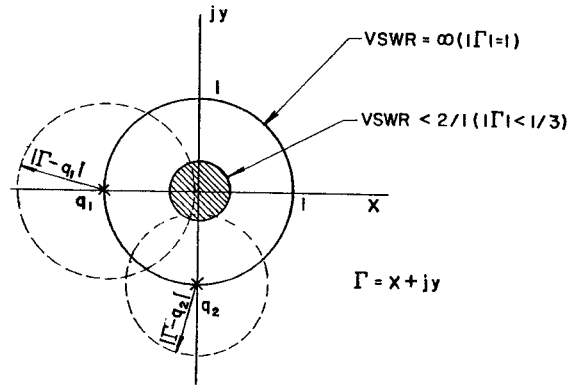


Fig.2 Optimum location of the points  $q_1, q_2$  in complex  $\Gamma$  plane for the measurement of small VSWR's with a five-port.

If a device can be constructed such that  $q_1 = -j$  and  $q_2 = -1$  and if the coupling to all detectors is the same, then it is readily shown that

$$\bar{P}_1 = P_1/P_{REF} = 1 + |\Gamma|^2 + 2|\Gamma| \cos \theta \quad (1)$$

$$\bar{P}_2 = P_2/P_{REF} = 1 + |\Gamma|^2 + 2|\Gamma| \sin \theta \quad (2)$$

where  $P_1, P_2$ , and  $P_{REF}$  are the powers measured at the three detectors. Equations (1) and (2) are readily solved for  $|\Gamma|^2$

$$|\Gamma|^2 = \frac{\bar{P}_1 + \bar{P}_2}{2} - \sqrt{\left(\frac{\bar{P}_1 + \bar{P}_2}{2}\right)^2 - \left(\frac{\bar{P}_1 - 1}{2}\right)^2 - \left(\frac{\bar{P}_2 - 1}{2}\right)^2} \quad (3)$$

We have coined the name resolver for the basic 4 port device of figure 1 for which equations (1) and (2) hold

to good approximation. No calibration would be necessary if the coupling to all detectors were the same. This is difficult to achieve to sufficient accuracy in practice, however. This difficulty can be overcome by making one reference measurement with a very good load (or sliding load) on the output. In this case  $\bar{P}_1$  and  $\bar{P}_2$  (expressed in dB) are given by the difference of the measured power levels and reference power levels with a load all in dB.

The fact that the measurement accuracy is independent of phase angle for small reflection coefficients can be demonstrated quite clearly by taking the logarithm of both sides of (1) and (2) and considering the limit  $|\Gamma| \ll 1$ . In this case,

$$\log_e \bar{P}_1 \approx \log_e (1 + 2|\Gamma| \cos \theta) \approx 2|\Gamma| \cos \theta$$

$$\log_e \bar{P}_2 \approx \log_e (1 + 2|\Gamma| \sin \theta) \approx 2|\Gamma| \sin \theta$$

Consequently, with a dual channel scope with logarithmic outputs and the memory subtraction feature (such as PMI's Model #1038) the two outputs will be directly proportional to the Smith chart variables in the limit of small  $|\Gamma|$ . This can be demonstrated by a) connecting the reference detector to the reference channel, detector 1 to channel A, detector 2 to channel B. b) storing a reference measurement with a good load in the memory of both channels. c) plotting the 2 outputs from the scope on an x-y recorder with the unit to be tested on the line.

#### A Practical Realization in Waveguide

The picture of a practical realization in WR-90 waveguide is shown in figure 3. It gives a good approximation to equations (1) and (2) over a waveguide band and looks something like a double cross guide coupler. The signal source connects to one of the main line ports and the test device to the other. The detectors connect to the cross arm outputs. Loads are inserted in the other ends of the cross arms. It can be shown that the location of the points  $q_1$ ,  $q_2$  on the edge of the unit circle of figure 2 places the following restrictions on the power division properties of this device:

- in each instance when feeding into a port, the power must divide equally out of the 2 ports of a coupled arm.
- in one case these coupled signals must be in phase and in the other case  $90^\circ$  out of phase.
- the coupling must be weak.

Weak coupling is necessary if the points  $q_1$  and  $q_2$  are to lie close to the unit circle. However if the coupling is too weak, then the signals will be noisy. A good compromise seems to be 30dB coupling. Such coupling is readily achieved with a single coupling hole. The resolver coupling is quite different from that of a directional coupler in that there is essentially no directivity. Power fed into the main arm will divide nearly equally from the two ports of a secondary arm.

Using the Bethe small hole theory, the conditions on the hole locations implied by conditions a) and b) can be determined. These will be given in a subsequent publication. Within the scope of this theory, condition a) can be satisfied at all frequencies. This is not the case for the  $90^\circ$  phase requirement of b). However, the Bethe theory predicts that the slope of the phase difference will be zero at a frequency within the waveguide band. As a result, the phase deviation

from  $90^\circ$  can be expected to be small over a waveguide band. More will be said on this point later.

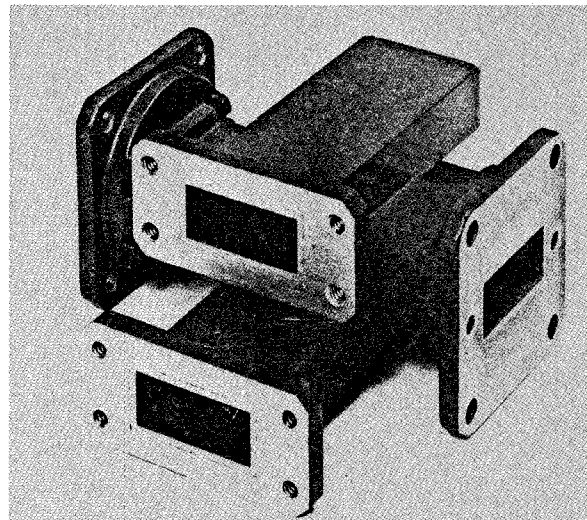


Fig.3 The picture of a WR-90 waveguide resolver

A coupling level of 30 dB has some other advantages. The 2 cross arms are well decoupled from one another. For a typical input power of 10mW, the output power to the detectors will be -20 dBm. This is at the upper end of the square law region for the most crystal detectors. The detected voltage will be on the order of 10 mV which is readily measurable to approximately  $1 \mu V$  accuracy with present technology.

#### Measurement of Some Standard Mismatches

Our basic power measurement unit is the PMI Model #1038 scope. The reference detector is connected to the reference channel, detector 1 to channel A, and detector 2 to channel B. It is operated in the A/ref and B/ref mode with both channels on the 1 dB/cm scale. This fairly high resolution scale is used because one is essentially measuring small deviations from a particular reference power level. This should be clear from equations (1) and (2). If the signal deviates by more than  $\pm$  full scale (i.e.  $\pm 4$  dB), then the VSWR  $> 2/1$ . This means that the reflection coefficient lies outside of the hatched circle of figure 2 and inaccuracies (even ambiguities) may result. The outputs from the scope go to an analogue to digital interface box. The digitized signals from this box are then fed into a minicomputer.

Table 1 gives the results of the measurement of a 1.50 VSWR standard, a 1.05 VSWR standard, followed by a re-measurement of the sliding load reference over the WR-90 waveguide band. The last column indicates that our system gives a typical error of .001 on repeating a VSWR reading. In addition there is an error of up to 3% for the measurement of large VSWR's. It is worth emphasizing that the great majority of time spent making these measurements is attributable to the mechanical operations of connecting the test device on or off the line, sliding loads etc.. Computation time is a small fraction of this.

At this point a confession should be made. As was mentioned previously, the Bethe small hole theory predicts that the  $90^\circ$  phase difference required by condition b) will be well satisfied over a waveguide band. In practice we have found that this difference slopes by about  $10^\circ$  over such a band. This would be a major source of error unless compensated for in the computer program. This is easily done and the results of Table

I include such a phase error compensation. If such an error is present, then (2) should be rewritten as

$$\bar{P}_2 = P_2/P_{REF} = 1 + |\Gamma|^2 + 2|\Gamma|\cos(\theta - 90^\circ + \Delta) \quad (4)$$

where the deviation  $\Delta$  varies nearly linearly over a waveguide band. Expanding the cosine term and using (4) in conjunction with (1), an expression for  $|\Gamma|^2$  similar to that given by (3) but containing  $\cos \Delta$  and  $\sin \Delta$  terms can be obtained.

#### A Technique for Measuring Large Reflection

##### Coefficients

It is apparent from figure 2 that  $\Gamma$ 's lying close to the line running from  $q_1$  to  $q_2$  can't be accurately measured as things stand. There is a simple way out of this difficulty however. If a precision rotary vane attenuator is installed between the output and device under test, then the effect of additional attenuation will be to move the points  $q_1$  and  $q_2$  away from the unit circle along the negative  $x$  and  $y$  axes respectively. For instance if 6 dB of attenuation is inserted, then  $q_1$  and  $q_2$  will lie on a circle four times as large in diameter as previously and the hatched area will now enclose the entire unit circle. Of course the sensitivity for measuring small  $\Gamma$ 's will also be four times poorer.

##### Conclusions

A theory for the optimum five port network has been developed using six port concepts. A practical realization in waveguide has been described. Particularly in waveguide the five port network promises to yield a compact structure which can be calibrated using a very simple calibration procedure.

##### References

1. Glenn F. Engen, "An Improved Circuit for Implementing the Six-Port Technique of Microwave Measurements", IEEE Trans. on MTT, Vol. MTT-25, pp. 1080-1083, December 1977.
2. Glenn F. Engen, "The Six-Port Reflectometer: An Alternate Network Analyser", IEEE Trans. on MTT, Vol. MTT-25, pp. 1075-1080, December 1977.

Table 1

Measurements of 1.50 and 1.05 VSWR standards with WR-90 waveguide resolver.

<u>FREQ.(MHz)</u>	<u>1.50 STANDARD</u>	<u>1.05 STANDARD</u>	<u>RE-REF.</u>
8200	1.479	1.052	1.002
8400	1.481	1.050	1.001
8600	1.497	1.048	1.002
8800	1.471	1.050	1.001
9000	1.497	1.052	1.001
9200	1.481	1.053	1.001
9400	1.491	1.053	1.001
9600	1.498	1.049	1.003
9800	1.496	1.047	1.001
10000	1.509	1.049	1.001
10200	1.495	1.050	1.001
10400	1.508	1.052	1.000
10600	1.502	1.050	1.001
10800	1.511	1.047	1.002
11000	1.501	1.047	1.001
11200	1.500	1.046	1.001
11400	1.495	1.049	1.002
11600	1.483	1.051	1.001
11800	1.510	1.050	1.001
12000	1.489	1.052	1.001
12200	1.498	1.051	1.001